Faraday Rotation of Microwave Background Polarization by a Primordial Magnetic Field

Arthur Kosowsky[1,2] and Abraham Loeb[1]
[1]Center for Astrophysics, Harvard University, 60 Garden St., Cambridge MA 02138
[2]Department of Physics, Lyman Laboratory, Harvard University, Cambridge, MA 02138

ABSTRACT

The existence of a primordial magnetic field at the last scattering surface may induce a measurable Faraday rotation in the polarization of the cosmic microwave background. We calculate the magnitude of this effect by evolving the radiative transfer equations for the microwave background polarization through the epoch of last scatter, in the presence of a magnetic field. For a primordial field amplitude corresponding to a present value of 10^{-9}G (which would account for the observed galactic field if it were frozen in the pre-galactic plasma), we find a rotation angle of around 1° at a frequency of 30 GHz. The statistical detection of this signal is feasible with future maps of the microwave background.

Subject headings: magnetic fields–cosmology:theory–cosmic microwave background

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1. Introduction

The origin of the observed μG magnetic fields in spiral galaxies has been a long-standing puzzle for more than three decades (Rees 1987; Kronberg 1994). On the one hand, these magnetic fields could have resulted from an exponential dynamo amplification of a small seed field with an e-fold period of a galactic rotation (Parker 1979; Zel'dovich, Ruzmaikin, & Sokoloff 1983; Field 1994). Alternatively, these fields may have already been frozen in the primordial plasma before galaxies formed (Hoyle 1958; Piddington 1964, 1972; Ohki et al. 1964), and could have even affected the evolution of structure in the universe (Wasserman 1978; Kim, Olinto, & Rosner 1994). The origin of the primordial field is still a subject of speculation (see, e.g. Turner & Widrow 1988; Quashnock, Loeb, & Spergel 1989; Vachaspati 1991; Ratra 1992).

In the past, various indirect theoretical arguments were used to favor the dynamo amplification mechanism over the primordial origin alternative (Zel'dovich, Ruzmaikin, & Sokoloff 1983). However, recent theoretical studies argue that a galactic dynamo should saturate due to the rapid growth of a fluctuating small-scale field before it can actually result in a coherent large-scale field of the type observed in galactic disks (Kulsrud & Anderson 1992; Vainshtein & Cattaneo 1992; Vainshtein, Parker, & Rosner 1993; Cattaneo 1994). The view that the galactic field may, in fact, be primordial gains additional support from observations of damped Lyα absorption systems in QSO spectra at $z_{\rm abs} \sim 2$. These systems, which are thought to be the progenitors of galactic disks (Lanzetta et al. 1995), add Faraday rotation to their background QSOs, consistent with them having μG fields (Welter, Perry, & Kronberg 1984; Wolfe, Lanzetta, & Oren 1992; Kronberg, Perry, & Zukowski 1992). Since these systems exist at an epoch when their rotation period is not exceedingly small compared to their age, this result could pose a problem to the standard dynamo hypothesis. However, the current data may not be sufficient to draw any firm statistical conclusions about absorption systems beyond a redshift of 0.4 (Perry, Watson, & Kronberg 1993).

The potential existence of a primordial magnetic field is also consistent with observations of clusters of galaxies. Faraday rotation measurements of radio sources inside and behind clusters indicate strong magnetic fields in many of them (Kim et al. 1990, 1991; Taylor & Perley 1993). The detected cluster fields have a typical magnitude of a few μG and a coherence length of 10^{1-2} kpc. The cores of several clusters contain tangled magnetic fields with amplitudes as high as $\sim 10^{1-2}\mu$G (Dreher et al. 1987; Perley & Taylor 1991; Taylor & Perley 1993; Ge & Owen 1993). In the outer halos of clusters, lower limits $\gtrsim 0.1\mu$G were set on the field amplitude, by combining measurements of synchrotron radio-emission from relativistic electrons in these halos together with lower limits on the associated hard X-ray emission due to Comptonization of the microwave background (Rephaeli 1979; Rephaeli et al. 1987; Rephaeli, & Gruber 1988).

If primordial in origin, the μG galactic field could have resulted directly from the adiabatic compression of a cosmological field, $B_0 \sim (10^{-10}$–$10^{-9})$G. Using a sample of 309 galaxies and quasars with a small intrinsic rotation measure, Vallee (1990) was able to set an upper limit of $10^{-9}\mathrm{G} \times (\Omega_{\rm IGM} h/0.01)^{-1}$ on the magnitude of a cosmological magnetic field which is coherent on the scale of the horizon; here, $\Omega_{\rm IGM}$ is the ratio between the ionized gas density in the intergalactic medium and the critical density, and h is the Hubble constant H_0 in units of 100 km s^{-1} Mpc^{-1}. However, this limit is based on Faraday–rotation measures, for which the contributions of field reversals along the line of sight average-out in a tangled field configuration. The limit is weakened to a value $\gtrsim 3 \times 10^{-8}\mathrm{G} \times (\Omega_{\rm IGM} h/0.01)^{-1}$ if the cosmic field is coherent only on scales $\lesssim 10$Mpc (see also Kronberg 1994).

In this paper, we propose a direct empirical probe of primordial magnetic fields. It is by now established that the microwave background should have acquired a measurable level of polarization at decoupling (see, e.g. Kosowsky 1996; Coulson, Crittenden, & Turok 1995). If a primordial magnetic field is present at the last scattering surface of the microwave photons, it will cause Faraday rotation of the direction of linear polarization. Since the rotation angle depends on wavelength, it is possible to infer this effect by comparing the polarization vector of the microwave sky in a given direction at two different frequencies. Since both the magnetic field amplitude ($\propto [1+z]^2$) and the baryonic density ($\propto [1+z]^3$) increase rapidly with redshift, this effect could potentially have measurable consequences at the high redshift of decoupling.

We can roughly estimate the expected rotation angle of the microwave background polarization as follows. Monochromatic radiation of frequency ν passing through a plasma in the presence of a magnetic field $\mathbf{B}$ along the propagation direction $\hat{\mathbf{q}}$ will have its linear polarization vector rotated at the rate

$$\frac{d\varphi}{dt} = \frac{e^3 x_e n_e}{2\pi m^2 \nu^2} (\mathbf{B} \cdot \hat{\mathbf{q}}), \tag{1}$$

where e and m are the electron charge and mass, n_e is the total number density of electrons, and x_e is the ionization fraction. (Throughout the paper we use natural units with $\hbar = c = G = 1$.) The optical depth for scattering is of order unity out to the redshift of decoupling when polarization is generated, i.e. we may substitute $\int x_e n_e dt \approx 1/\sigma_T$ where σ_T is the Thomson cross-section. The rms rotation angle can then be easily estimated from the time–integral of equation (1) by noticing that B/ν^2 is time–independent and by averaging $\varphi^2 \propto (\mathbf{B} \cdot \hat{\mathbf{q}})^2$ over all possible orientations of $\mathbf{B}$,

$$\langle \varphi^2 \rangle^{1/2} \approx \frac{e^3 B_0}{2\sqrt{2}\pi m^2 \sigma_T \nu_0^2} = 1.6^\circ \left(\frac{B_0}{10^{-9}\mathrm{Gauss}} \right) \left(\frac{30\ \mathrm{GHz}}{\nu_0} \right)^2, \tag{2}$$

where B_0 is the current amplitude of the cosmological magnetic field, and ν_0 is the observed frequency of the radiation. Note that equation (2) is independent of cosmological parameters.

For a primordial field of 10^{-9} G which could result in the observed galactic field, we therefore expect a rotation measure of order 1.6 deg cm^{-2} = 280 rad m^{-2}. This rotation is considerable by astrophysical standards and could in principle be measured. The exact value of the rotation measure is, however, sensitive to the growth history of the microwave background polarization through the surface of last scatter. A larger Faraday rotation is expected in anisotropic cosmological models, previously investigated by Milaneschi & Fabbri (1985).

In this paper, we perform a detailed calculation of the above Faraday rotation signal. We work in the context of inflation-type models with adiabatic initial fluctuations, and assume no early reionization. The rotation generated at the surface of last scatter depends only on the ionization history and is insensitive to most cosmological parameters. Section 2 develops the formalism for calculating the microwave background polarization including Faraday rotation. In Section 3 we present numerical results, including the dependence of the rotation angle on the mean mass density and baryon density of the universe. Finally, Section 4 discusses the prospects for detecting this effect.

2. Formalism

The evolution of the cosmic microwave background is described by a set of radiative transfer equations for the Fourier modes of the radiation brightnesses $\Delta_I(\mathbf{k}, \hat{\mathbf{q}}, \eta)$, $\Delta_Q(\mathbf{k}, \hat{\mathbf{q}}, \eta)$, and $\Delta_U(\mathbf{k}, \hat{\mathbf{q}}, \eta)$, where the subscripts refer to the standard Stokes parameters, the Fourier mode wavevector is given by $\mathbf{k}$, the radiation propagation direction is given by the unit vector $\hat{\mathbf{q}}$, and $\eta = \int(1+z)dt$ is conformal time. For pure blackbody fluctuations, the temperature deviation is $\Delta T/T_0 = \Delta_I/4$, with $T_0 = 2.726 \pm 0.010$ K (Mather et al. 1993) the mean temperature. In the case of no magnetic fields, the transport equations depend only on $\mu = \cos(\hat{\mathbf{k}} \cdot \hat{\mathbf{q}})$, the cosine of the angle between a given Fourier mode and the propagation direction. Also, the isotropy of the Universe implies that the equations depend only on $k = |\mathbf{k}|$ and not on the direction of the Fourier component. In the following calculation, we assume a uniform magnetic field and extract mean results by averaging over the entire sky; then the equations still depend only on k and μ and not on $\hat{\mathbf{q}}$ and $\mathbf{k}$ separately. This is equivalent to assuming that the magnetic field is coherent on the scale of the width of the last scattering surface, a comoving scale of about 5 Mpc. This assumption is natural if the coherent magnetic field observed in galaxies came from a primordial origin, since galaxies were assembled from a comoving scale of a few Mpc.

Including the Faraday mixing term, the radiative transport equations in comoving coordinates are (Bond & Efstathiou 1984; Kosowsky 1996)

$$\dot{\Delta}_I + ik\mu(\Delta_I - 4\Phi) \;\; = \;\; -4\dot{\Psi} - \dot{\tau}\left[\Delta_I - \Delta_{I0} + 4v_b\mu - \frac{1}{2}P_2(\mu)(\Delta_{I2} + \Delta_{Q2} - \Delta_{Q0})\right]$$

$$\begin{aligned}
\dot{\Delta}_Q + ik\mu\Delta_Q &= -\dot{\tau}\left[\Delta_Q + \frac{1}{2}\left(1 - P_2(\mu)\right)\left(\Delta_{I\,2} + \Delta_{Q\,2} - \Delta_{Q\,0}\right)\right] + 2\omega_{\mathbf{B}}\Delta_U \\
\dot{\Delta}_U + ik\mu\Delta_U &= -\dot{\tau}\Delta_U - 2\omega_{\mathbf{B}}\Delta_Q,
\end{aligned} \tag{3}$$

where Ψ and Φ are scalar metric perturbations in the Newtonian gauge, v_b is the baryon velocity, and $\dot{\tau}$ is the differential optical depth, defined by $\dot{\tau} = x_e n_e \sigma_T a/a_0$ with x_e the ionization fraction, n_e the total electron density, σ_T the Thomson cross-section, and a the scale factor normalized to a_0 today. Dots over quantities represent derivatives with respect to conformal time η. The numerical subscripts on the radiation brightnesses indicate moments defined by an expansion of the directional dependence in Legendre polynomials $P_\ell(\mu)$:

$$\Delta_{I\,\ell}(k) \equiv \frac{1}{2}\int_{-1}^{1} d\mu\, P_\ell(\mu)\Delta_I(k,\mu). \tag{4}$$

A detailed derivation of equations (3) can be found in Kosowsky (1996).

The mixing terms between Δ_Q and Δ_U account for the effect of Faraday rotation; these terms follow directly from the definition of the Stokes parameters. The conformal Faraday rotation rate is given by

$$\omega_{\mathbf{B}} \equiv \frac{d\varphi}{d\eta} = \frac{d\varphi}{dt}\frac{a}{a_0}, \tag{5}$$

where φ is the rotation angle of the polarization vector. For a given magnetic field and direction of photon propagation, the rotation rate $d\varphi/dt$ is given by equation (1). The time dependence of the gravitational potentials Φ and Ψ and the ionization fraction x_e then completely determine the evolution of the radiation brightnesses.

Hu and Sugiyama (1995a,b) have demonstrated how to solve the radiative transport equations semi-analytically, using analytic fits to the evolution of the potentials and the ionization fraction. This formalism has recently been extended to include polarization (Zaldarriaga & Harari 1995). Essentially, the tight-coupling solution describing the primordial plasma is modified by a damping factor to account for diffusion damping through recombination, and the resulting photon distribution free-streams to the present epoch. We use this approach to give the temperature fluctuations, which then source the polarization fluctuations. Here we present the modifications to the tight-coupling solution necessary to incorporate Faraday rotation; for other details of the calculation, see Hu & Sugiyama (1995a,b).

The polarization brightness Δ_Q is sourced by the function $S_p \equiv \Delta_{I\,2} + \Delta_{Q\,2} - \Delta_{Q\,0}$, and Δ_U is generated as Δ_Q and Δ_U are rotated into each other. In the absence of magnetic fields, Δ_U retains its tight-coupling value of zero. Expanding to second order in the tight-coupling parameter $\dot{\tau}^{-1}$ gives the evolution equation (Zaldarriaga & Harari 1995)

$$\dot{S}_p + \frac{3}{10}\dot{\tau}S_p \approx \frac{2}{5}ik\Delta_{I\,1}, \tag{6}$$

with the solution

$$S_p(k,\eta) \approx \frac{2}{5} ike^{3\tau(\eta)/10} \int_0^\eta d\eta'\, e^{-3\tau(\eta')/10} \Delta_{I\,1}(k,\eta'). \tag{7}$$

Note that the total optical depth τ is defined as

$$\tau(\eta) = \int_\eta^{\eta_\star} \dot{\tau}(\eta) d\eta, \tag{8}$$

with $\eta_\star$ the conformal time of recombination, giving $d\tau/d\eta = -\dot{\tau}$. We can rewrite the equations for the polarization brightnesses in a simple form with the change of variables

$$\begin{aligned} \tilde{\Delta}_Q &\equiv e^{ik\mu\eta-\tau}\Delta_Q, \\ \tilde{\Delta}_U &\equiv e^{ik\mu\eta-\tau}\Delta_U, \\ \tilde{S}_p &\equiv e^{ik\mu\eta-\tau} S_p, \end{aligned} \tag{9}$$

giving the evolution equations

$$\begin{aligned} \dot{\tilde{\Delta}}_Q(k,\mu,\eta) &= -\frac{3}{4}\dot{\tau}(\eta)(1-\mu^2)\tilde{S}_p(k,\eta) + 2\omega_{\mathbf{B}}(\mu,\eta)\tilde{\Delta}_U(k,\mu,\eta), \\ \dot{\tilde{\Delta}}_U(k,\mu,\eta) &= -2\omega_{\mathbf{B}}(\mu,\eta)\tilde{\Delta}_Q(k,\mu,\eta). \end{aligned} \tag{10}$$

We are interested in the case of small Faraday rotation; the first-order iterative solution to equations (10) is

$$\begin{aligned} \tilde{\Delta}_Q(k,\mu,\eta) &= -\frac{3}{4}(1-\mu^2)\int_0^\eta d\eta'\dot{\tau}(\eta')e^{ik\mu\eta'-\tau(\eta')}S_p(k,\eta'), \\ \tilde{\Delta}_U(k,\mu,\eta) &= \frac{3e^3(\mathbf{B}\cdot\hat{\mathbf{q}})}{4\pi m^2 c\nu^2\sigma_T}(1-\mu^2)\int_0^\eta d\eta'\dot{\tau}(\eta')\int_0^{\eta'} d\eta''\dot{\tau}(\eta'')e^{ik\mu\eta''-\tau(\eta'')}S_p(k,\eta''). \end{aligned} \tag{11}$$

Equations (7) and (11) determine the polarization of the radiation as a function of frequency, given the ionization history x_e and the temperature dipole brightness $\Delta_{I\,1}$.

Using the polarization brightnesses in k-space, we next obtain an expression for the polarization vector in x-space. Before performing the Fourier integral, the various spherical coordinate systems defined for each $\mathbf{k}$-mode must be rotated to a common system since the definitions of the Stokes parameters Q and U depend on the orientation of the coordinate system (for details, see Kosowsky 1996). The real-space polarization fluctuations are given by

$$\begin{aligned} \frac{Q(\mathbf{x},\theta,\phi)}{T_0} &= \frac{1}{4}\sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}}\left[\Delta_Q(\mathbf{k},\theta',\phi')\cos 2\xi' + \Delta_U(\mathbf{k},\theta',\phi')\sin 2\xi'\right], \\ \frac{U(\mathbf{x},\theta,\phi)}{T_0} &= \frac{1}{4}\sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}}\left[-\Delta_Q(\mathbf{k},\theta',\phi')\sin 2\xi' + \Delta_U(\mathbf{k},\theta',\phi')\cos 2\xi'\right], \end{aligned} \tag{12}$$

where (θ', ϕ') represents the same direction as (θ, ϕ) except in the coordinate system defined by the mode k. The angle ξ' is the rotation angle to align the coordinate systems; an expression for ξ' is given in Kosowsky (1996) but will not be used in what follows. Finally, the polarization vector $\mathbf{P}$ can be constructed from these quantities as

$$\mathbf{P}(\mathbf{x}, \theta, \phi) = \frac{1}{\sqrt{2}}\left[\hat{\theta}\sqrt{P(P+Q)} + \hat{\phi}\frac{U}{|U|}\sqrt{P(P-Q)}\right], \tag{13}$$

with $P \equiv \sqrt{Q^2 + U^2}$. The angle between two polarization vectors φ_{12} follows from

$$\cos^2\varphi_{12} = \frac{(\mathbf{P}_1 \cdot \mathbf{P}_2)^2}{P_1^2 P_2^2} = \frac{1}{2}\left[1 + \frac{Q_1 Q_2 + U_1 U_2}{P_1 P_2}\right]. \tag{14}$$

The predictions of a cosmological model are only statistical in nature. For a given magnetic field, the observationally interesting quantity is the expectation value of the Faraday rotation angle. Calculationally, it is useful to consider only averages of quantities quadratic in the brightnesses given above; we thus consider the observable

$$\langle\cos^2\varphi_{12}\rangle \approx \frac{1}{2}\left[1 + \frac{\langle Q_1 Q_2 + U_1 U_2\rangle}{\langle P_1^2\rangle^{1/2}\langle P_2^2\rangle^{1/2}}\right]. \tag{15}$$

The averages can be calculated explicitly in terms of the brightnesses by replacing the averages with the integral $V^{-1}\int d\mathbf{x}$ and replacing the Fourier sums with integrals, $\sum_{\mathbf{k}} \to [V/(2\pi)^3]\int d\mathbf{k}$, where V is a volume normalization factor. We also include an exponential beam suppression to account for a Gaussian beam of width σ (Kolb & Turner 1990). The necessary averages can be written in terms of the two integrals

$$\begin{aligned} I_Q &\equiv \int_{-1}^{1} d\mu \int_0^\infty k^2 dk \exp[-4k^2 H_0^{-2}(1-\mu^2)\sigma^2]|\Delta_Q(k,\mu)|^2, \\ I_U &\equiv \int_{-1}^{1} d\mu \int_0^\infty k^2 dk \exp[-4k^2 H_0^{-2}(1-\mu^2)\sigma^2]|\Delta_U(k,\mu)|^2 \end{aligned} \tag{16}$$

with H_0 the Hubble constant; a long calculation gives the final equations

$$\begin{aligned} \langle Q_1 Q_2 + U_1 U_2\rangle &= \frac{V}{64\pi^2}(I_Q + f I_U) \\ \langle P_1^2\rangle &= \frac{V}{64\pi^2}(I_Q + I_U) \\ \langle P_2^2\rangle &= \frac{V}{64\pi^2}(I_Q + f^2 I_U), \end{aligned} \tag{17}$$

where the subscripts 1 and 2 refer to two different frequencies $\nu_1 < \nu_2$, all of the brightnesses are evaluated at ν_1, and $f \equiv \nu_1^2/\nu_2^2$. Since we are comparing the polarization at two frequencies in the same direction on the sky, the formulas look like the temperature correlation function at zero separation.

In the perturbative limit of small rotation angles, equation (15) reduces to

$$\langle\varphi_{12}^2\rangle^{1/2} \approx \frac{1}{2}\left(1-\frac{\nu_1^2}{\nu_2^2}\right)\left(\frac{I_U}{I_Q}\right)^{1/2}. \tag{18}$$

Equations (11), (16), and (18) imply that the rms rotation angle is proportional to the magnetic field and inversely proportional to the square of the frequency, as expected. An average over all directions of observation will reduce the above estimate by a factor of $\sqrt{2}$, due to the changing orientation of the magnetic field.

3. Results

We solve the equations in the previous section numerically to determine I_U/I_Q and thus the proportionality factor between the mean Faraday rotation signal and the quantity B/ν^2. Normally, the radiative transfer equations for the microwave background are expanded in a moment hierarchy to eliminate the μ dependence; however, in this case, the additional directional dependence of the magnetic field direction complicates the moment expansion. We perform the integrals in equations (11) on a (μ,k) grid. The necessary cosmological inputs are the dipole component of the radiation intensity, Δ_{I1}, and the differential optical depth through recombination, $\dot{\tau}$.

We choose as an illustrative model "standard" cold dark matter, but the nature of the perturbations has little effect on our results; the size of the radiation dipole and the details of recombination affect the final answer, but neither of these depends strongly on the cosmological model. The matter density $\Omega_0 h^2$ and the baryon density $\Omega_b h^2$ have a mild effect on recombination, which we include through numerical evolution of the free electron density. Hu and Sugiyama (1995a) give an analytic approximation for Δ_{I1} which we use here. We evolve the electron density using a numerical code incorporating the recombination physics detailed in Hu et al. (1995).

Figure 1 displays the evolution of the polarization brightnesses Δ_Q and Δ_U through the last scattering surface for $\mu = 0.5$ along with the differential visibility function $\dot{\tau}e^{-\tau}$. At early times, the tight coupling between the photons and the baryons prevents the development of polarization. As decoupling proceeds, the photons begin to free-stream, generating a quadrupole perturbation which sources the polarization. However, the induced Faraday rotation depends on the free electron density, which drops to negligible values as recombination ends. Rotation is generated during the brief period of time when the free electron density has dropped enough to end tight coupling but not so much that Faraday rotation ceases. Figure 1 shows that the generation of rotation lags behind the generation

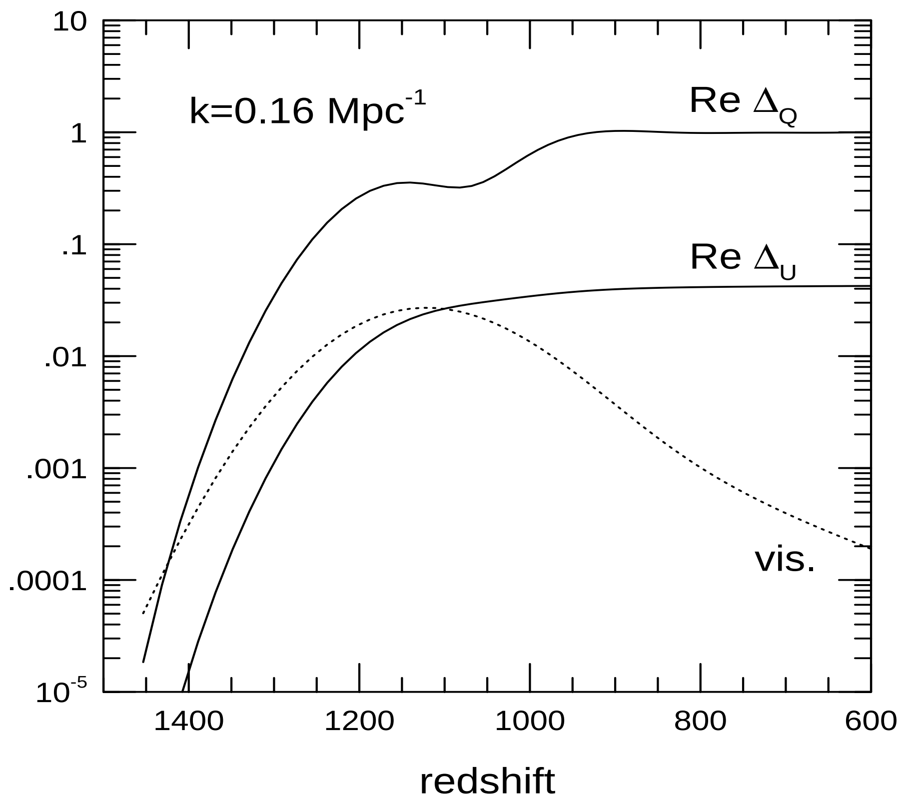

Fig. 1.— The evolution of the polarization brightnesses, for $k = 0.16\,\mathrm{Mpc}^{-1}$ and $\mu = 0.5$ (in arbitrary units). Also plotted as a dotted line is the differential visibility function $\dot{\tau}e^{-\tau}$ in units of Mpc^{-1}.

of polarization, and that rotation is essentially completed by $z = 1100$ whereas polarization generation continues almost until $z = 900$.

Figure 2 displays I_U/I_Q in equation (18) as a function of the cosmological parameters $\Omega_0 h^2$, the mass density parameter, and $\Omega_b h^2$, the baryon density parameter, for $B_0 = 10^{-9}$ G, $\nu_1 = 30$ GHz, and a beam-width of $0.5°$. The beam-width only affects the signal-to-noise ratio of the polarization signal and has virtually no effect on the size of the rotation angle. As expected in the Introduction, the value of $\langle\varphi^2\rangle \propto I_U/I_Q$ is nearly independent of these cosmological parameters. To within a 10% correction for the effects of $\Omega_b h^2$ and $\Omega_0 h^2$ (in the range $\Omega_b h^2 > 0.007$ and $\Omega_0 h^2 < 0.3$), the rotation angle is

$$\langle\varphi_{12}^2\rangle^{1/2} = 1.1^\circ \left(1 - \frac{\nu_1^2}{\nu_2^2}\right)\left(\frac{B_0}{10^{-9}\,\mathrm{G}}\right)\left(\frac{30\,\mathrm{GHz}}{\nu_1}\right)^2, \tag{19}$$

in good agreement with equation (2). We have included here a factor of $1/\sqrt{2}$ due to an average over all orientations of the magnetic field.

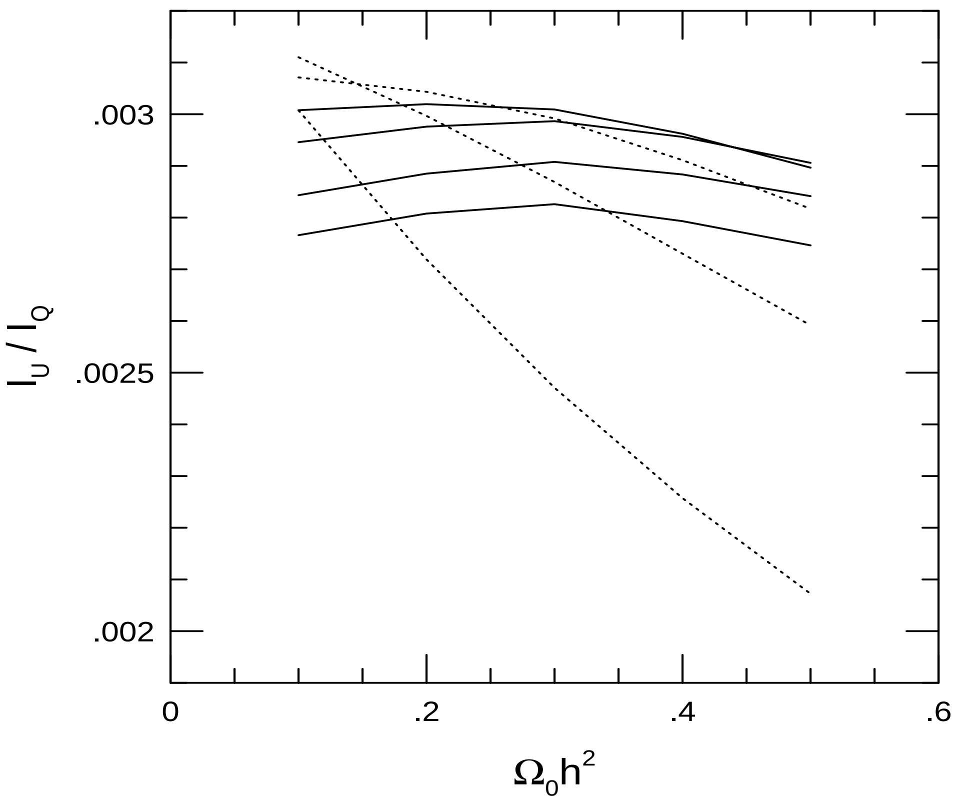

Fig. 2.— The ratio I_Q/I_U as a function of $\Omega_0 h^2$ and $\Omega_b h^2$, for a magnetic field $B_0 = 10^{-9}$ G and an observed frequency of 30 GHz. The dotted lines from bottom to top are for $\Omega_b h^2 = 0.005$, 0.0075, and 0.01. The solid lines from top to bottom are for $\Omega_b h^2 = 0.0125$, 0.015, 0.02, and 0.025.

4. Discussion

We have calculated the rotation of the microwave background polarization vector due to a primordial magnetic field at the surface of last scatter. The magnetic field was assumed to be coherent on the width of the last scattering surface, corresponding to a comoving scale of a few Mpc, which is conveniently the length scale from where galaxies are assembled. We have found that the mean Faraday rotation has a size of about 1° for a cosmological magnetic field of $B_0 = 10^{-9}$ G and an observed frequency of 30 GHz. An optimized experiment could measure the orientation of the polarization vector at one frequency which is as low as practicable, and at a second, somewhat higher frequency. If the frequency ratio is 2, then 75% of the total rotation is observed between the two frequencies. While extracting this rotation in a single observation direction is unlikely, a statistical detection averaged over many observation directions is possible.

The peak amplitude of the expected polarization fluctuations is an order of magnitude lower than that of the microwave temperature fluctuations, of order 10^{-6} (see, e.g., Crittenden, Davis, & Steinhardt 1993; Frewin, Polnarev, & Coles 1994). To measure a Faraday rotation of 1° requires another factor of 10^2 in sensitivity. The total sensitivity to the Faraday rotation signal is proportional to the raw pixel sensitivity and to the square root of the number of pixels. Currently envisioned satellite experiments are designed with the primary goal of mapping the microwave sky temperature down to small angular scales; given amplifiers of a certain sensitivity, it is advantageous to design an experiment with a signal-to-noise ratio per pixel of around unity and as many pixels on the sky as possible (Knox 1995). Such a mapping experiment would require of order 10^6 pixels to detect a rotation angle of 1°. This corresponds to a beam size of order $10'$ which is at the limit of current design proposals. It is not unreasonable to expect that in the coming years, raw sensitivity will improve so that systematic effects become the dominant obstacle to detecting the Faraday rotation signal. As with temperature measurements, the ultimate limitation will be contamination from foreground sources. While little is known about polarization sources at microwave frequencies, estimates suggest that in the frequency window of 30–80 GHz the signal should be dominated by the microwave background contribution (Timbie 1995). Experiments which focus on high signal-to-noise observations of small patches of the sky may also prove useful; their feasibility could be first demonstrated by searching for a foreground rotation signal in the direction of galaxy clusters with detected rotation measures from radio source observations (see, e.g. Dreher et al. 1987; Perley & Taylor 1991; Taylor & Perley 1993; Ge & Owen 1993). In fact, microwave background polarization measurements at multiple frequencies can potentially map the magnetic field distribution in cluster environments where the rotation measure is already known to be large (Loeb & Kosowsky 1996).

Will foreground contamination of the rotation measure be a considerable obstacle to measuring the Faraday rotation signal from primordial recombination? If the universe was reionized at a redshift $z \ll 100 \times \Omega^{1/3}(\Omega_b h/0.02)^{-2/3}$, then the optical depth through reionization is much smaller than unity and the additional Faraday rotation contribution is small. The contribution from late reionization would be further reduced if the coherence length of the primordial field is much smaller than a Gpc. In fact, the cumulative rotation measure of the intergalactic medium out to $z \approx 3$ is known to be smaller than a few rad m^{-2} from samples of high-redshift objects (see Valee 1990, Kronberg 1994, and references therein). In addition, the rotation measure of the Milky-Way galaxy was measured at high latitudes and found to be $\lesssim 20$ rad m^{-2} (Spitzer 1978; Simard-Normandin & Kronberg 1980). According to equation (19), these foregrounds should allow the detection of a cosmological field as small as 10^{-10} G.

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